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Fluid distribution and transport in porous media at low wetting phase saturations

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Abstract. Capillary pressure, hydraulic conductivity and the capillary dispersion coefficient have been observed to obey power laws in the wetting phase saturation. We relate power-law behaviour at low wetting phase saturations, i.e. at high capillary pressures, to the thin-film physics of the wetting phase and the fractal character of the pore space of natural porous media.

1. Introduction

The capillary pressure and the hydraulic conductivities (permeabilities) of wetting and non-wetting phases in a porous medium are complicated functions of wetting phase saturation S_w, the morphology and composition of pore space, and the process whereby wetting and non-wetting phases come to occupy the porous medium. The capillary pressure and permeability behaviour at low saturations of wetting phase provides information about the asperities or roughness of pore space and about the physics of thin films on pore surfaces that cannot be obtained from the high saturation studies. The properties of wetting phase (water) at low saturation is of substantial interest to the soil scientist in connection with agricultural crops, the balance of aquifers and the migration of agricultural, chemical and nuclear contaminants.

In this paper we describe recent work [1,2] in which we have derived and tested scaling laws based on the hypotheses that pore space asperities or roughness are fractally distributed and that at sufficiently low saturation the hydraulic conductance of wetting phase is controlled by thin films.

2. Porespace morphology of natural sandstone

A scanning electron micrograph of a fracture-exposed surface of an oil reservoir sandstone is shown in figure 1 [3]. The major grains and pores of this clayey consolidated sandstone are on the order of 100 micrometres in dimension (figure 1(a)). Stacks of book-like kaolinate clays (figure 1(b), (e), (f)) are attached to the pore surfaces. These stacks sometimes extend hundreds of micrometres in width. The spacing between the booklets is about 10 micrometres and between the kaolinate platelets is less than a micrometre (figure 1(f)). There are also aggregates of quartz crystallites (chert) on the pore surfaces (figure 1(c) and (d)). The crystallites are a few tenths of a micrometre in

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SURFACE TEXTURE OF RESERVOIR SANDSTONE

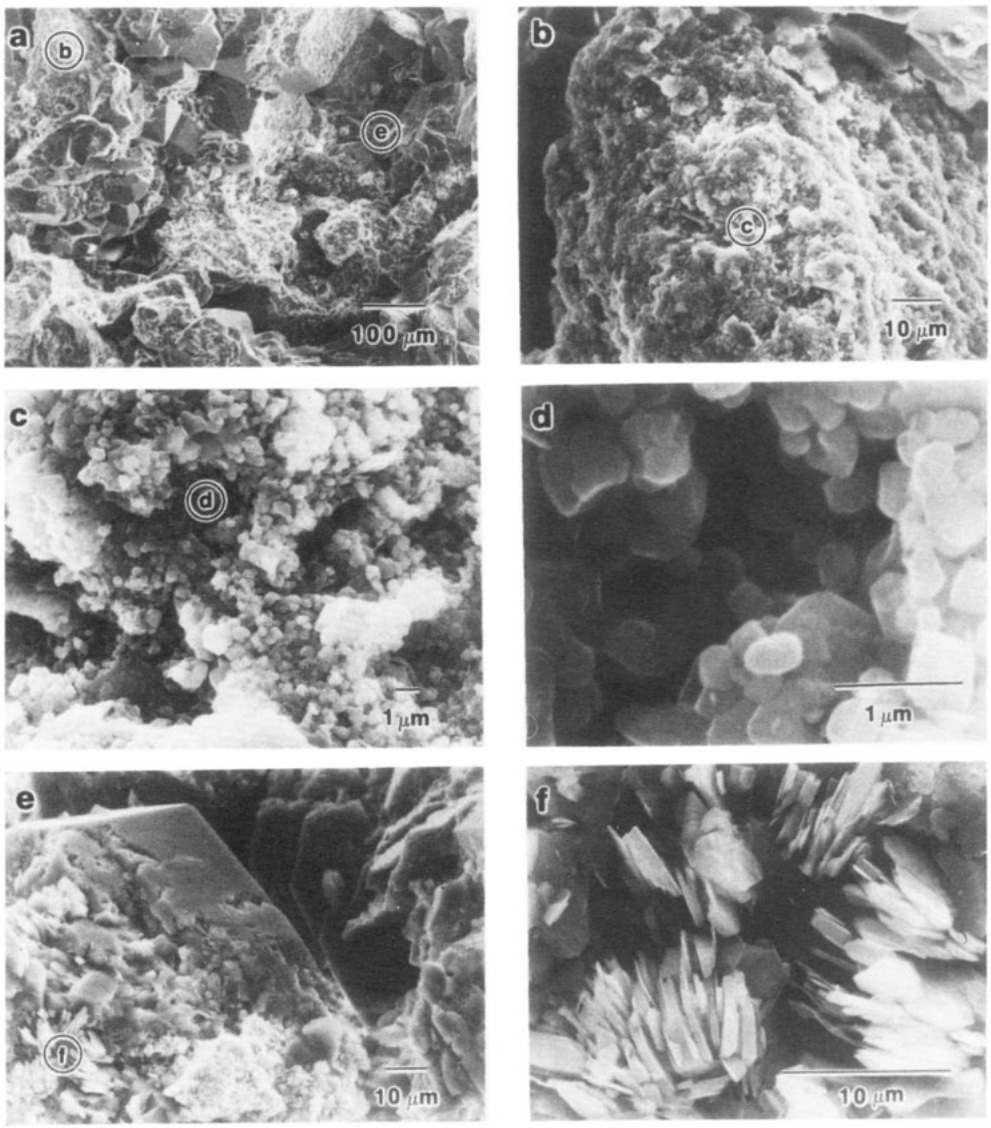

Figure 1. Scanning electron micrograph exposing the texture of the porosity of reservoir sandstone [3].

diameter. Between the aggregates are porous regions of the order of a few micrometres to a micrometre in diameter. Within the chert aggregate are intergranular spaces of the order of a few hundredths of a micrometre in size.

The multisized objects in figure 1 are consistent with scanning electron microscope (SEM) studies reported for several natural sandstones by Katz and Thompson [4]. They measured the number of features (asperities) versus size of the feature on several natural porous sandstone fracture surfaces and concluded that pore space is fractally distributed between a minimum dimension l_1 (of the order of 10 to 100 Å) and a

maximum dimension l_2 (of the order of 100μm). According to the argument of Katz and Thompson the porosity ϕ of fractal sandstone is

$$\phi = A(l_1/l_2)^{3-D} \tag{1}$$

where A is of the order of 1 and D is the fractal or Hausdorff dimension. By plotting the number of geometric features against size, they found values of D ranging from 2.57 to 2.87 from SEM studies of five natural sandstones.

If the volumes of the cavities formed by the various asperities illustrated in figure 1 are fractally distributed, this has particular implications for the capillary pressure behaviour at low saturations of wetting phase. In what follows we explain and report tests of these implications.

3. The capillary pressure and the saturation of pendular structures and thin films

We assume one of the fluids strongly wets the porous medium. Thus, even at saturations so low that bulk wetting phase seemingly exists only as isolated regions or pendular structures, the wetting phase remains hydraulically connected through thin films. To achieve a given wetting phase saturation S_w, the pressures P_w in bulk wetting phase and P_{nw} in bulk non-wetting phase must satisfy the Young–Laplace (YL) equation for the capillary pressure P_c, i.e. $P_c \equiv P_{nw} - P_w = 2H\gamma$, where H is the mean curvature and γ is the interfacial tension of the meniscus between wetting and non-wetting phases. The mean curvature is related to the principal radii of curvature R_1 and R_2 of the meniscus by the expression $2H = (1/R_1 + 1/R_2)$. Fixing the mean curvature of the menisci between wetting and non-wetting phases fixes the saturation S_w.

The Young–Laplace equation is applicable when both the wetting and non-wetting phases are present in bulk amounts. When the wetting phase is present as a thin film (thinner than a few thousand angstroms), the thin film pressure differs from that of bulk wetting phase by the disjoining pressure $\Pi(h)$ which depends on film thickness h. In this case the Young–Laplace equation must be replaced with the augmented Young–Laplace (AYL) equation [5]

$$P_c = \Pi(h) + 2H\gamma. \tag{2}$$

The YL and AYL equations provide the basis for understanding how the capillary pressure sets the inventory of wetting fluid at saturations below the percolation threshold, in which case wetting phase exists as thin films or as 'pendular structures' at intergranular contacts, i.e. isolated regions of smaller-than-average porosity, or in nooks and crannies provided by the pore surface asperities. The pendular structures are hydraulically connected by thin films. In conical and hemispherical pits, the menisci of the pendular structures are pieces of a sphere whose radius r_c ($H = r_c^{-1}$) is given by the YL equation $r_c = 2\gamma/P_c$, whereas the menisci along the pore edges and between the fused spheres are pieces of a cylinder whose radius r_c ($H = (2r_c)^{-1}$) is $r_c = \gamma/P_c$. By elementary geometry, it follows that the volume of wetting phase in pits scales as r_c^3 and in wedges and collars scales as r_c^2. Thus, since pits, pore edges and fused grains are the most likely sites of pendular structures, we expect the scaling

relation $S_{ps} = AP_c^{-2} + BP_c^{-3}$ if the sites are non-fractal, i.e. if they are all of the same characteristic length scale. If, on the other hand, the pendular structures are in sites fractally distributed according to (1), then at a capillary pressure P_c all the porosity with dimension $l < r_c = 2\gamma/P_c$ would be filled with wetting phase and so $S_{ps} \propto r_c^{3-D}$, or

$$S_{ps} = AP_c^{-(3-D)}. \tag{3}$$

Pendular structures in self-similar fractal media are depicted in figure 2. Shortly after the appearance of the work of Katz and Thompson, de Gennes [6] pointed out that (3) holds in the cases of a pore space of self-similar iterative flocs of the type shown in figure 2 and a pore space of self-similar iterative pits (self-similar pits within pits). The Menger sponge shown in figure 2 is an interesting example because its fractal dimension can be easily determined. The sponge is generated by the following iterative process. A cube with sides of length R is partitioned into 27 cubes the length of whose sides is $R/3$. The seven cubes along the three axes of symmetry of the parent cube are removed. The partitioning and deletion process is iterated for each of the remaining cubes. The Menger sponge is made by indefinite continuation of this process. The resolution length r is defined as the size of the smallest measurable feature. The number N_r of solid cubes increases with resolution as $N_r = (R/r)^D$ and the fractal or Hausdorff dimension D of the sponge equals $\log 20/\log 3 \simeq 2.73$.

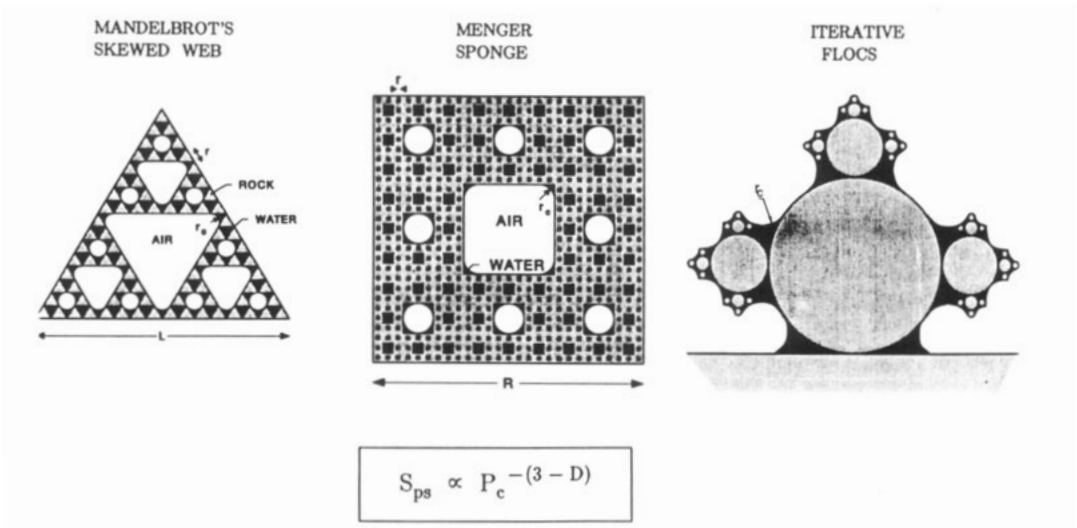

Figure 2. Wetting liquid present in fractally distributed pendular strutures. The examples given here are geometrically regular, self-similar fractal objects.

As is obvious from figure 1, a natural sandstone is not a regular self-similar fractal. Instead, consistent with the arguments given by Katz and Thompson [7], we believe it is better viewed as a disordered assembly of matrix grains ranging in size from l_{1g} to l_{2g}. The size distribution of pores will naturally be broad if that of the granular subparts is broad. From the work of Katz and Thompson we anticipate $D \simeq 2.5-2.8$,

and so according to (3) the pendular structure inventory approaches zero as $P_c^{-0.5}$ to $P_c^{-0.2}$, a much weaker function of P_c than in the case of non-fractal structures.

At saturations below the wetting phase percolation threshold bulk wetting phase does not span the sample. Instead, wetting phase is present as isolated pendular structures hydraulically connected to one another through thin films. The thickness of these films is fixed by the capillary pressure through the AYL equation, equation (2). Thin films tend to form in regions of pore space whose mean radius of curvature is large compared to film thickness h, that is in relatively smooth regions. Otherwise, pendular structures will form instead of thin films. Thus, we expect the capillary contribution, $2H\gamma$, to be negligible compared to the disjoining pressure contribution, i.e. we expect $P_c \simeq \Pi(h)$ for thin films.. If the disjoining pressure obeys a power law of the form $\Pi(h) \propto h^{-m}$, it then follows from $P_c \simeq \Pi(h)$ that $S_{tf} \propto P_c^{-1/m}$.

The total saturation of wetting phase is given by $S_w = S_{tf} + S_{ps}$. However, under the conditions of the experiments to be discussed in a later section, the thin film saturation is generally negligible compared to that of pendular structures, and so the behaviour of the capillary pressure against wetting phase saturation can be used as a probe of pore space roughness independently of the disjoining pressure behaviour.

4. Hydraulic conductivity and capillary dispersivity

The local flow of wetting phase at low saturations is very complicated because it involves flow in geometrically complex pendular structures and thin films. However, if the pendular structures do not span the sample, the major resistance to flow will be the thin films of thickness h. Thus, $K_w \propto h^3$, and if the disjoining pressure obeys the power law $\Pi(h) \propto h^{-m}$, then $K_w \propto P_c^{-3/m}$. If, furthermore, the inventory of wetting phase is primarily that of pendular structures fractally distributed, equation (4), the hydraulic conductivity obeys the scaling law $K_w \propto S_w^{3/m(3-D)}$. In this case, the saturation dependence of the hydraulic conductivity of wetting phase yields information regarding the disjoining pressure dependence on film thickness and the geometry of pore space.

If a sample porous medium at low wetting phase saturation contacts a reservoir of wetting phase, it will spontaneously imbibe wetting phase according to a convection-dispersion equation in which the capillary dispersivity, D_c, is [8] $D_c = -(K_w/\mu_w)(dP_c/dS_w)$. The effect of capillary dispersion can be either hypodispersive, diffusive or hyperdispersive depending on how $D_c(S_w)$ behaves with decreasing S_w. If $D_c(S_w)$ tends to zero as S_w approaches zero, the invading front of wetting phase disperses less than in a diffusive front. If $D_c(S_w)$ tends to infinity as S_w approaches zero, the invading front disperses more broadly than in a diffusive front. If $D_c(S_w)$ tends to a constant value as S_w approaches zero, the front in the experiment indicated spreads in a diffusive manner.

If $S_w \simeq S_{ps}$, if the pendular structures are fractally distributed, and if thin films control hydraulic conductivity, the capillary dispersion coefficient obeys the scaling law $D_c \propto S_w^{[3-m(4-D)]/m(3-D)}$ at low wetting phase saturations. Thus, the spreading will be hypodispersive if $m < 3/(4-D)$, hyperdispersive if $m > 3/(4-D)$ and diffusion-like if $m = 3/(4-D)$. By changing wetting phase (e.g., water in air against oil in air), one could expect to find both hypo and hyperdispersion in the same porous medium.

Bacri *et al* [8] observed hyperdispersion in oil/water flow in a porous medium. However, their experiments were conducted at saturations higher than those for which we expect the scaling laws given here to hold.

5. Comparison of theory and experiment

Few data exist on the behaviour of a wetting phase at low saturation in a porous medium. In the soils literature, scaling laws of the form $P_c \propto S_w^{-a}$, $K_w \propto S_w^b$ and $D_c \propto S_w^c$, where a, b and c are positive constants, have been observed empirically [9–13].

We located and analysed [1,2] three sets of carefully measured data on capillary pressure and hydraulic conductance of sandstones and clayey soils. Melrose [14] carried out a series of measurements of the capillary pressure of water in the presence of air in Berea sandstone. Davis [1] found that the capillary pressure curve obeys the scaling law $P_c \propto S_w^{-2.23}$, which implies a fractal roughness of the pore space of Berea sandstone with a fractal dimension $D = 2.55$. This result compares favourably with the range of values $2.57 < D < 2.87$ found by Katz and Thompson in SEM studies of other natural sandstones.

Nimmo and Akstin [15] reported measurements of the capillary pressure and hydraulic conductance of water at low saturations in the presence of air in several compacted samples of Oakley sands (a clayey soil). The data of Viani *et al* [16] on the disjoining pressure of clayey soils fit the formula $\Pi(h) = 250h^{-1/2}$dyn cm$^{-3/2}$). We analysed [2] their data in terms of the scaling laws $P_c \propto S_w^{-1/(3-D)}$ and $K_w \propto S_w^{3/m(3-D)}$ implied by the assumptions of negligible thin film inventory, fractally-distributed pendular structures, thin film controlled hydraulic conductivity, and a power law disjoining pressure $\Pi(h) \propto h^{-m}$. The capillary pressure data were used to obtain the fractal dimension D. We set $m = 1/2$ in accordance with the results of Viani *et al* and computed from $3/m(3-D)$ the exponent of the scaling law for K_w. The results are summarized in table 1. The fractal dimensions deduced from the capillary pressure curves range from 2.09 to 2.62 with an average value of 2.45. With the exception of sample 4, the fractal dimensions of all the samples compare well to the value 2.55 of Berea sandstone.

Table 1. Fractal dimension and permeability scaling law for compacted Oakley sands (data from [15]).

Sample	Fractal dimension D	Exponent of $K_w(S_w)$	
		$3/m(3-D)$	Measured
1	2.59	15.2	12.7
2	2.37	10.0	12.7
3	2.62	16.5	14.1
4	2.09	6.9	8.9
5	2.26	8.5	15.3
6	2.35	9.7	8.4
7	2.67	19.1	13.4
8	2.52	13.2	12.2
9	2.59	15.5	16.8

The predicted exponent for K_w agrees with the measured value to better than 24% for all samples except numbers 5 and 7. Given the simplicity of the model, the agreement between theory and experiment as demonstrated in table 1 is satisfactory.

As a matter of clarification, we point out that the assumption that film flow controls the hydraulic conductance does not require that most of the flow paths be thin films. For example, suppose the conductance of a strip of film is 1000 times less than that of a strip of the same length of a pendular structure. Then if in a series of strips, only 1% are thin films, the net hydraulic resistance of the thin films in the series is ten times larger than the net hydraulic resistance of the pendular structures in the series.

Ward and Morrow's [17] capillary pressure curves for water in the presence of air in a number of low permeability sandstones are plotted in figure 3. The curves have two distinct regions. There is a lowest saturation region where the deduced value of D is near 2 and there is a higher saturation region in which D ranges from 2.61 to 2.89. The values of D in the higher saturation region are consistent with the findings of Katz and Thompson, but those in the low saturation region (especially those less than 2) do not fit the fractal model. It is possible that in the low saturation regime (where the capillary pressures are greater than 100 bars, the highest capillary pressure in Melrose's experiment was 93.6 bars) the pendular structures have been totally removed and only thin films remain. If the disjoining pressure of the thin films obeys the water-on-quartz law, $\Pi(h) \propto h^{-1}$, then $S_{tf} \propto P_c^{-1}$, and if $S_w \simeq S_{tf}$, then the expected scaling law would be $P_c \propto S_w^{-1}$, consistent with the value $D = 2$. Thus, perhaps the low saturation region of the capillary pressure curves of figure 3 is one in which most of the wetting phase occupies thin films, and the higher saturation region is one in which the wetting phase samples a fractal distribution of pore sizes.

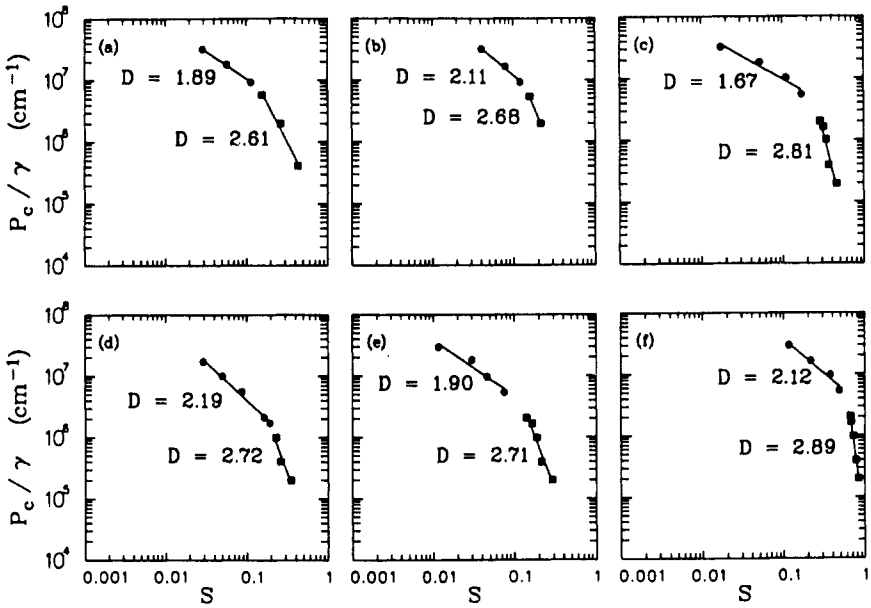

Figure 3. Capillary pressure curves for water in low permeability sandstones [17].

In summary, under the assumptions that pendular structures occupy pore space

of fractal dimension D, that the disjoining pressure obeys the power law $\Pi(h) \propto h^{-m}$, that the wetting phase inventory is primarily pendular structures, and that thin films control the hydraulic resistance of wetting phase, we deduce the power laws

$$X \propto S_w^{\nu_x} \qquad X = P_c,\ K_w,\ D_c \tag{4}$$

where for capillary pressure $\nu_x = -1/(3-D)$, for permeability $\nu_x = 3/m(3-D)$, and for capillary dispersion $\nu_x = [3 - m(4-D)]/m(3-D)$. The experimental work described above lends support to the scaling laws for P_c and K_w in the cases of natural sandstones and clayey soils.

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